Why inertial systems need to be aligned and how to do this effectively

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Inertial Navigation Systems (INS) have been gaining popularity in the past years, especially due to the miniaturization and the lower cost of new sensors developed for the automotive world. However, choosing and operating effectively an INS is not straightforward. One of the reasons is that before being operated, an INS needs to go through an initial process called alignment.

The aim of this paper is to explain what the alignment is, why it is necessary and how to perform the alignment effectively. It focuses on the physical principles underlying the performance, and not on the practical implementation, to help understand how the sensor performance impacts the system performance.

Since the number of inertial applications and parameters is almost infinite, this paper aims at providing some simple rules and formula that the reader will be able to adapt to their specific needs. In addition to choosing the best alignment method for a given INS, this will also help the reader to operate an INS more efficiently.

It is organized as follows:

- The first section explains what is an INS, and its differences with an IMU (Inertial Measurement Unit). The main constraints associated with their use will be introduced, especially why an INS needs to go through a specific procedure usually called "alignment" to estimate the initial condition of the system
- The second section details the various parameters that must be initialized during the alignment process, and how the sensor performance impacts their estimation
- Finally, the third and fourth sections focus on the heading determination: they explain why the heading is the most difficult parameter to estimate, and how this can be done based on the user constraints.

CONTENT

I. What is an inertial navigation system?

WHAT AN INS DOES

An IMU (Inertial Measurement Unit) is a set of gyros and accelerometers. Coupling an IMU with a rotation and acceleration integration algorithm would theoretically allow to compute the speed, position and orientation at any time, without relying on any external information. This unique peculiarity makes the IMU the perfect set of sensors for positioning, as it is the only technology that can guarantee to provide a measurement, no matter what.

However, things are not that simple, for two reasons:

- To integrate the rotation and acceleration, one needs to know the initial conditions, that means at least the initial position, speed and orientation
- IMU sensors are not perfect, so all the parameters computed using such system (position, speed and orientation) will drift over time

For those reasons, IMU data are usually not integrated indefinitely with time. They are fused with any possible information the user can get (speed, position, range to a known location, etc.) to compute the initial conditions and compensate for the drift.

This is what an Inertial Measurement System (INS) does: an INS is an IMU and an algorithm which aims at:

- determining the initial condition
- integrating the IMU measurements
- limiting the drift using external sensors, a hypothesis on the movements made by the vehicle and a so-called “data fusion” algorithm.

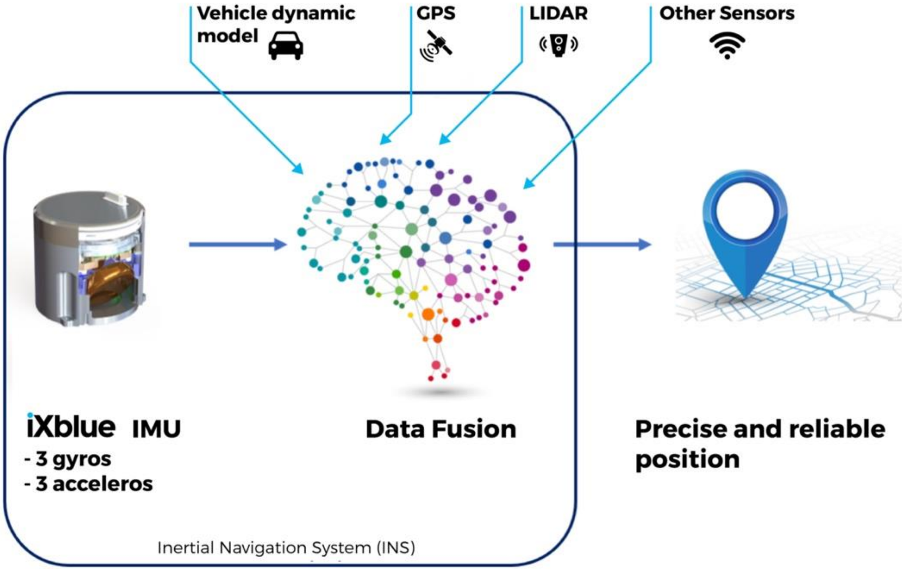

Figure 1: What is an INS?

The data fusion algorithm takes advantage of the correlation between different navigation parameters to correct various or all parameters with only one type of measurement. The theory of such filter is beyond the scope of this paper, but to give a simple example, a roll or pitch error will create a speed error, which in turn will create a position error. Correcting for the position will also correct the speed and the roll and pitch.

Various data fusion algorithms can be used, such as the particle filter, mixture of gaussian, etc. In real-life, most people use a version of the Kalman Filter (Extended Kalman Filter, Unscented Kalman Filter, etc.) because this class of filter is very robust, requires little processing power, and can be optimized upon some hypothesis that are usually met by an INS system.

FOCUS ON THE INERTIAL SENSORS

Inertial sensors are sensors that measure a motion (gyros: rotation, accelerometers: linear acceleration) with respect to an inertial frame, that means a static frame with respect to the stars, not the Earth.

As any sensor, they are never perfect. The main imperfection impacting the performance of inertial sensors are:

- Noise: Stochastic (random) error that cannot be estimated or compensated
- Bias: constant or slowly varying additive term.
- Scale factor error: constant or slowly varying multiplicative term

Those errors can be modelled as follows:

$$\mathrm{m} = (1 - \delta\mathrm{K}).\tilde{\mathrm{m}} - \mathrm{b} + \mathcal{N}(0, \sigma) \qquad \text{Eq. 1}$$

With:

- m: true value of the parameter being measured (rotation rate or acceleration)
- $\tilde{\mathrm{m}}$: measurement from the sensor
- $\delta\mathrm{K}$: sensor scale factor error
- b: Sensor bias
- $\mathcal{N}(0, \sigma)$: White noise with 0 mean and standard deviation: σ

The sensor misalignments, which are defined as constant or slowly varying error on the measurement axis, have been omitted from that list because they usually have a low impact on the system performance.

In addition, bias and scale factor errors can be split into two categories:

- Bias or scale factor error repeatability: Value of the parameter when the system is switched on. It is considered constant as long as the system is on. It can often be estimated and compensated.
- Bias or scale factor error stability: Variation of the parameter over time while the system is on. The error instability is generally an order of magnitude lower than the repeatability.

<u>Notice</u>: To simplify the analysis, this document will often take the example of an INS perfectly orientated with the North, West and Vertical directions. This is extremely useful to analyze the effects that depends on geographical factors. Consequently, sensor biases will often be referred to as "East" or "North" bias, for instance.
In reality, this situation never occurs. However, since the East or North bias is a linear combination of true sensor biases (that depends on the orientation of the INS), the conclusion can be generalized to any situation.

II. How to determine the initial conditions?

As explained before, the initial conditions are at least the initial position, the initial speed and the initial orientation (heading, roll, pitch) of the system. However, additional parameters sometimes need to be estimated before navigating, or before reaching the full performance of the system, such as the gyros or accelerometer biases, scale factors, or other parameters related to external sensors.

WHY IS IT IMPORTANT TO DETERMINE THE INITIAL CONDITIONS?

Determining the initial conditions is a process often called "alignment" or sometimes "calibration"[1]. Usually, this alignment period has 2 objectives:

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[1] The term « calibration » is most often used to define the process by which some installation parameters, such as the GNSS antenna lever arms (coordinates of GNSS antenna in the INS reference frame) or the speed sensor misalignment with respect to the INS, are estimated. It is thus important, when using those terms, to define what it means, as there is no standard convention accepted by the whole community.

- Determining initial conditions: Determining the position, speed and orientation is required before integrating the gyros and accelerometer (as explained above)
- Linearizing the system: Most navigation data fusion filters make some gaussian assumptions, meaning that the various navigation parameters errors have linear relationship between them. Since the orientation is defined by three angles (H: heading, R: roll and P: pitch), they must be determined accurately enough to make this assumption true, which means that their error can be linearized accurately enough to the first order, i.e.:
  - $\sin(\delta H[\mathrm{rad}]) \approx \delta H\ [\mathrm{rad}]$ (same thing for roll/pitch)
  - $\cos(\delta H[\mathrm{rad}]) \approx 1$ (same thing for roll/pitch)

Notice: the letter δ before any parameter defines an error associated with the parameter. For instance, δH is the heading error.

The accuracy required for such an assumption to be correct depends on the system performance required: the more accurate the angle, the more accurate the 1st order approximation, the least artificial noise is brought into the system.

A rule of thumb is that an error of 0.1 rad (6 deg) is acceptable as a starting value if correctly mitigated in the algorithm.

POSITION

The initial position needs to be provided by the user, because the INS cannot determine this position by itself. This is usually done using a GNSS, or manually when the user knows its initial position.

In some cases, the latitude can be determined by the INS, if the sensors are accurate enough. However, this requires very high-end sensors, some practical constraints, and does not provide very good results. The basic physical principle behind latitude estimation is that the angle between the gravity (measured by the accelerometers) and the Earth rotation (measured by the gyros) depends on the Latitude [Ref4].

The longitude, on the other hand, cannot be determined by the INS, as it has no physical existence (it is just a convention).

SPEED

The speed cannot be determined by the accelerometers only. Once again, the accelerometers can be integrated, but the initial speed needs to be known.

The main method to determine the initial speed is to use a GNSS (Global Navigation Satellite System).

When this is not possible (for underground applications for instance), the system is usually required to start with a null speed, or to use a speed sensor, such as an odometer or a DVL (Doppler Velocity Log) for subsea applications.

ROLL / PITCH

As explained before, there are two reasons to determine the initial roll and pitch values:

- Reaching a sufficient accuracy to make some first order simplification
- Making sure the system operates at its full performance

The roll/pitch determination is usually a 2-step process:

- An initial algorithm is used to reach the first order linearization constraint
- Then the standard data fusion algorithm is used to improve the accuracy

Reaching the first order approximation validity

As explained before, it is often required to know an angle value with an accuracy of up to 6° before making any linear assumption. The reason for that is that 6° is about 0.1 rad, so a first order approximation will result in neglecting terms of order 2 (and more), that means in the order of magnitude of 0.1^2=1%.

When the system is static, perfect accelerometers would measure only the gravity. Since perfect accelerometers do not exist, they measure the gravity and some errors, mainly the bias (Figure 2).

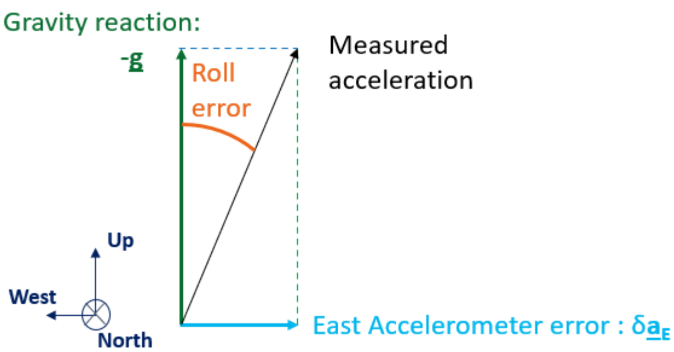

Figure 2: Impact of an accelerometer error on the roll error

The relationship between the horizontal accelerometer error and the roll and pitch errors can be deduced from Figure 2. To the first order:

$$\delta\mathrm{RP[rad]} = \frac{\delta \mathrm{a}}{\mathrm{g}} \qquad \text{Eq. 2}$$

With:

- $\delta\mathrm{RP}$: [rad] Roll or pitch error
- $\delta\mathrm{a}$: [m/sec^2] Accelerometer error
- g: [m/sec^2] Gravity (g~9.81 m/sec^2)

The accelerometer error is mainly due to the accelerometer bias repeatability. It is then useful to use the former equation to evaluate the impact of an accelerometer bias on the roll and pitch performance:

$$\delta\mathrm{RP[rad]} = \frac{\mathrm{b^a}}{\mathrm{g}} \qquad \text{Eq. 3}$$

With:

- δRP: [rad] Roll or pitch error
- b^a: [m/sec^2] Accelerometer bias
- g: [m/sec^2] Gravity (g~9.81 m/sec^2)

A 1mG accelerometer bias will lead to a 1mrad (~0.06 deg) roll/pitch error. The 6° target is consequently easily reached.

In addition, the same reasoning can be applied if the roll and pitch error are computed considering that the true acceleration is 0, when it is in fact not zero.

In that case:

$$\delta RP[rad] = \frac{a_H}{g} \qquad \text{Eq. 4}$$

With:

- $a_H : [m/sec^2]$: Horizontal acceleration

Since g is about 10m/sec^2, a roll or pitch accuracy better than 0.1 rad, can only be achieved for horizontal acceleration lower than 1m/sec^2.

When a system is accelerating, this acceleration threshold can be reached, but there are very few practical cases when alignment is necessary while the system is accelerating.

Getting the full performance

The previous paragraph explained that, in static conditions, the roll and pitch performance depend on the bias at startup: the bias repeatability.

However, if some movements are possible, using an external speed or position sensor, or using multiple static positions, then the roll pitch accuracy do not depend on the bias repeatability, but on the bias stability, since the bias repeatability can be rapidly estimated by the Kalman filter. The bias stability is usually an order of magnitude lower than the bias repeatability, and that can make a significant difference.

This can be demonstrated using a simple theoretical example, with a system kept static at 2 consecutive positions:

- the first position of the system is 0° Heading
- Then the system is turned around the Z axis by 180° so that the second position is 180° Heading

For the first position, a positive accelerometer bias will create a Western speed error, and for the second position, a positive accelerometer bias will create an Eastern speed error (see Figure 3).

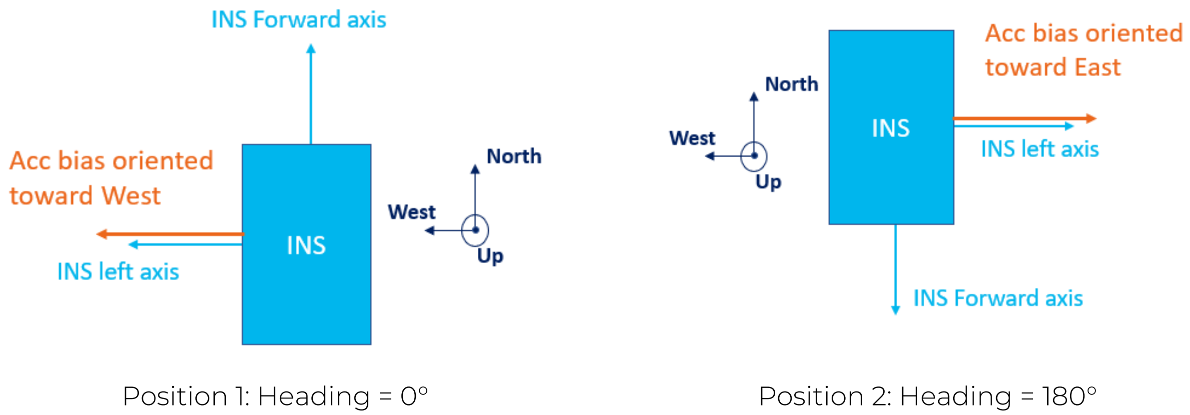

Figure 3: 2-position static alignment

As the true acceleration is 0 in both cases, it leads to the following system, with two equations and two unknowns:

$$\begin{cases} \int (\delta R.G + b_L^a).dt = \int \delta acc^{Pos1}.dt \\ \int (\delta R.G - b_L^a).dt = \int \delta acc^{Pos2}.dt \end{cases} \qquad \text{Eq. 5}$$

With:

- δR: [rad] Roll error
- G: [m/sec²] Gravity
- b_L^a: [m/sec²] Total bias (repeatability + stability) of the left accelerometer
- δacc^{Pos1}, δacc^{Pos2}: [m/sec²] INS acceleration error. The integral of the error is the speed computed by the INS, since the true speed is 0. Consequently $\int \delta acc^{Pos1}.dt$ and $\int \delta acc^{Pos2}.dt$ are at the same time the speed error and the speed measured by the INS respectively at position 1 and 2.

This system, with 2 equations and 2 unknown parameters δR and b_L^a, can easily be solved since the INS provides the speed error at both static positions. However, since the bias is not constant, the best estimate of the roll will depend on the accelerometer bias stabilities.

In the general case, when moving and using an external sensor, the principle is the same: you can have multiple observations of a weighted sum of the roll error and East/West velocity error. In that case the speed is not null, but you know its value (using a speed sensor) or its integrated value (using a position sensor). So the same rule applies: in non-static conditions, the roll/pitch error can be determined using the bias accelerometer instability:

$$\delta RP[rad] = \frac{b^{ai}}{g} \qquad \text{Eq. 6}$$

With

- δRP: [rad] Roll or pitch error
- b^{ai}: [m/sec²] Accelerometer bias instability
- g: [m/sec²] Gravity (g~9.81 m/sec²)

How long does it take to measure the roll and pitch?

In reality, the accelerometer bias is not the only error: the accelerometer measurements are also impacted by the VRW (Velocity random walk) due to the accelerometer white noise. Since the integration of a white noise is proportional to the square root value of the time, we can write:

$$\begin{cases} \delta V^{noise} = \mathrm{VRW} \times \sqrt{t} \\ \delta V^{bias} = b^a \times t \end{cases} \qquad \text{Eq. 7}$$

With:

- δV^{noise}: [m/sec] Speed error due to the VRW
- δV^{bias}: [m/sec] Speed error due to the accelerometer bias
- VRW: [$m/sec/\sqrt{Hz}$] Accelerometer velocity random walk
- b_a: [m/sec^2] Accelerometer bias
- t: [sec] time

The bias can be estimated when the error it creates on the speed becomes higher than the speed error due to the noise, that means when:

$$b^a \times t > \mathrm{VRW} \times \sqrt{t} \qquad \text{Eq. 8}$$

Or:

$$t > \left(\frac{\mathrm{VRW}}{b^a}\right)^2 \qquad \text{Eq. 9}$$

Numerical application: The IXAL-A5-50 [Ref1] has a white noise of $10\mu G/\sqrt{Hz}$ (translates to a VRW= 10^{-4} $m/sec/\sqrt{Hz}$) and a bias stability of 1mG which means: 0.01m/sec^2).

The accelerometer bias (and so the roll and pitch) can then be estimated after averaging the data for

$$t = \left(\frac{10^{-4}}{0.01}\right)^2 = 10^{-4}\mathrm{sec}$$

Since this value is smaller than the usual sampling rate, it means the accelerometer noise can be averaged instantaneously for such sensor.

Synthesis

Because roll and pitch are angles, they need to be estimated roughly before a standard, linear navigation filter such as the Kalman filter can be used.

The standard method to do so is to consider the system static and determine them from the gravity measurements.

The two main factors affecting the initial roll and pitch estimation are the accelerometer biases and the system displacement, but both effects can be cancelled out using various static positions.

HEADING

The heading is the most difficult parameter to estimate by an INS. Various methods exist to determine the heading, each with varying advantages and limitations. This section will examine those methods.

In the literature, many techniques are proposed to determine the initial heading. However, they all relate to one of the 4 following categories described shortly below, and in more details afterwards.

Measuring the Earth rotation using only the inertial sensors

By definition the heading is the angle between the forward axis of the system and the Earth rotation rate projected in the horizontal plane, as illustrated in Figure 4.

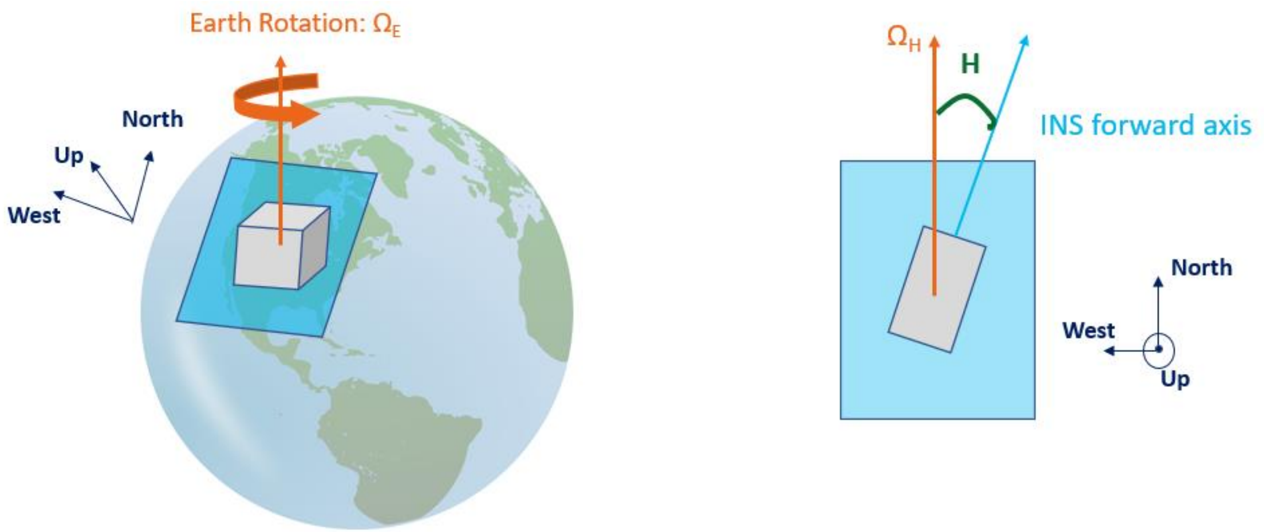

Figure 4: Heading definition: The heading is the angle between the forward axis and the North direction, defined as the Earth rotation vector projected in the horizontal frame

The Earth rotation rate projected in the horizontal plane is defined by (See Figure 5):

$$\Omega_H = \Omega_E . \cos(\mathrm{Lat}) \qquad \text{Eq. 10}$$

With:

- Ω_H: Earth rotation rate projected in the horizontal plane
- Ω_E: Earth rotation rate
- Lat: Latitude

Since the projection of the Earth rotation in the horizontal plane is zero at the poles, the heading does not mean anything when one is located at the North or South Pole.

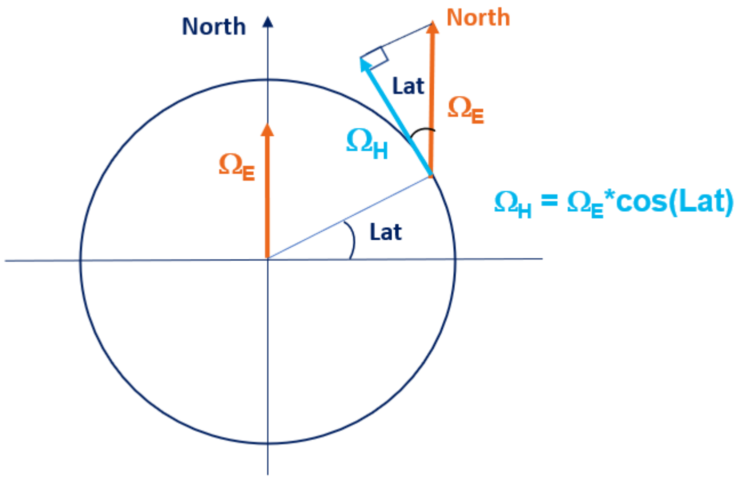

Figure 5: Projection of the Earth rotation rate in the horizontal frame

The simplest way to determine the heading is thus the measurement of the rotation in the horizontal plane. However, the Earth rotation is very small: 15,04°/h, which means 0,004°/sec. Only very accurate sensors can measure it! And they have to be very static. Alternatively, as will be explained below, the derivative of the gravity can be used to measure the Earth rotation: very accurate sensors are still mandatory, but the static constraints can be removed.

The gyro accuracy required to determine the heading makes it THE critical parameter of an INS. The ability to find and maintain it makes the most important difference between two categories of INS.

Measuring the magnetic field

It is well known that the magnetic field is roughly directed in the South/North direction, so using a magnetic sensor is an alternative to determine the initial heading. In this case, a map should be used to compensate for the difference between the magnetic field and the Geographic North.

Using an external geographical information

If the INS is coupled with a sensor able to find a heading using geographical information (such as a double antenna GNSS or a perceptive sensor), the device can communicate the heading to the INS to initialize it.

Measuring the displacements in the geographical frame

By comparing the movements of the INS (measured by the IMU) and a positioning sensor such as a GNSS, it is possible to determine the heading of the INS. In that case, it will be shown that the heading accuracy mainly depends on the accelerometers and the accuracy of the external sensor.

III. Heading determination using the inertial sensors

As explained in the previous section, the heading can be determined by measuring the Earth rotation in the horizontal plane (c.f. Figure 4). To do so, the gyro measurement or the acceleration derivative can be used. In this section, we detail how those two methods can work and what their interests and limits are.

USING THE GYROS

How it works – theory in the perfect case

Measuring the heading can be done directly using the gyro measurements if:

- The angle between the horizontal plane and the INS is known (this is the case if the roll and pitch have already been measured using the methods described above)
- AND the system is completely still. In that case, the rotation vector measured by the gyros is directly the Earth rotation vector.

This method can be illustrated using a perfectly horizontal INS.

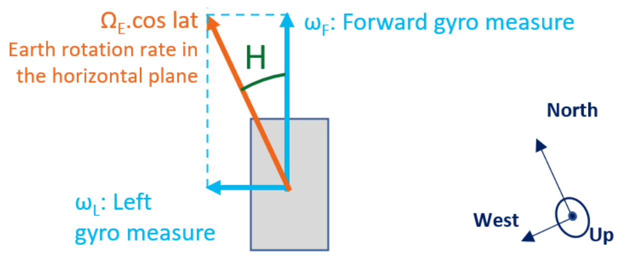

Figure 6: Measuring the Earth rotation to determine the heading

It can be deduced from Figure 6 that:

$$\mathrm{H[rad]} = \tan^{-1}\left(\frac{\omega_L}{\omega_F}\right) \quad \text{Eq. 11}$$

Performance in static conditions

Gyros are never perfect, so there is always an error on the rotation measurement. If the gyros are used to determine the heading, those imperfection will translate into a heading error. The following figure helps understanding how a gyro error will impact the heading.

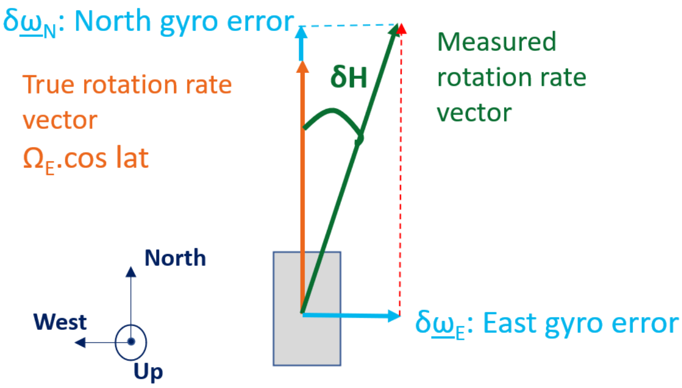

Figure 7: Impact on a gyro bias on the heading estimation

From Figure 7, the heading error can be written as:

$$\delta\mathrm{H} = \sin^{-1}\frac{\delta\omega_\mathrm{E}}{\Omega_\mathrm{E}.\cos(\mathrm{lat}) + \delta\omega_\mathrm{N}} \qquad \text{Eq. 12}$$

With:

- $\delta\mathrm{H}$: [rad]
- $\delta\omega_\mathrm{E}$: [°/h] East component of the rotation vector error measured by the gyros (due to gyros imperfections)
- $\delta\omega_\mathrm{N}$: [°/h] North component of the rotation vector error measured by the gyros (due to gyros imperfections)
- Ω_E: [°/h] Earth rotation rate. $\Omega_\mathrm{E} = 15.04\ °/\mathrm{h}$
- lat: latitude

To be able to determine the heading, the gyro error must be significantly lower than the Earth rotation, which is 15.04°/h (see Ref3). Otherwise, the Earth rotation vector cannot be measured. Consequently, this method only applies in the case of small gyro errors with respect to the Earth, and the former equation can be linearized to the first order in $\delta\omega_\mathrm{E}$ and $\delta\omega_\mathrm{N}$.

$$\delta\mathrm{H}[\mathrm{rad}] \approx \frac{\delta\omega_\mathrm{E}}{\Omega_\mathrm{E}.\cos(\mathrm{lat})} \qquad \text{Eq. 13}$$

The heading error can be converted from radians to degrees:

$$\delta\mathrm{H}[°] \approx \frac{180}{\pi}\frac{\delta\omega_\mathrm{E}}{\Omega_\mathrm{E}.\cos(\mathrm{lat})} \qquad \text{Eq. 14}$$

This can be approximate by:

$$\delta\mathrm{H}[°\mathrm{seclat}] \approx 4.\delta\omega_\mathrm{E} \qquad \text{Eq. 15}$$

With:

- $\delta\mathrm{H}[°\mathrm{seclat}]$: is the heading accuracy in degrees seclat (the heading accuracy at a given latitude is found by dividing the seclat value by the cosine of the latitude)

- $\delta\omega_E$: [deg/h] East gyro bias.

Notice that the error $\delta\omega_E$ is mainly due to the gyro bias:

- Since the Earth rotation rate is null in the East direction, the scale factor error has no impact
- The noise can be averaged with time

So, in true static condition, the heading accuracy of an INS which determines the heading directly from the gyro measurements will be:

Impact of the gyro bias repeatability on the heading precision in static condition	
$\delta H[°\mathrm{seclat}] \approx 4.\, b_E^{gr}[°/h]$	Eq. 16

With:

- $\delta H[°\mathrm{seclat}]$: is the heading accuracy in seclat degrees
- b_E^{gr}: [°/h] East gyro bias repeatability

Notice the seclat term, which means that the error must be divided by the cosine of the latitude. This implies that the closer to the Pole, the less accurate the heading; and that the heading cannot be computed at the pole.

Based on those results it is clear that one needs to rely on sensors with a very low bias repeatability to find the north. Because the orientation of the INS during the alignment is not known, one needs 3 accurate gyros (or at least 2 if it is known that the INS will be horizontal during the alignment).

INS that are able to measure the Earth rotation are called “gyrocompassing”, or “North finding”, because they have the ability to determine the heading without the help of an external sensor.

A commonly accepted rule of thumb is that an INS is North finding if its gyros have a bias repeatability of 0.1°/h or lower (which means a heading accuracy of 0.4°seclat).

Performance with multiple positions

If the system can be configured in multiple static positions, the reasoning used for the roll/pitch (see paragraph “Getting the full performance”) can be reapplied directly. In that case, the heading accuracy does not depend on the bias repeatability, but on the bias instability:

Impact of the gyro bias instability on the heading precision in multi-static condition	
$\delta H[°\mathrm{seclat}] \approx 4.\, b_E^{gi}[°/h]$	Eq. 17

With:

- δH: $[°\mathrm{seclat}]$ Heading accuracy
- b_E^{gi}: [°/h] East gyro bias instability

Because those sensors require an external operation (movement by 180°), they are not considered North finding.

Performance when the system is moving North

A North speed will create a rotation in the West-East direction, due to the ellipsoidal shape of the Earth. This is illustrated in Figure 8.

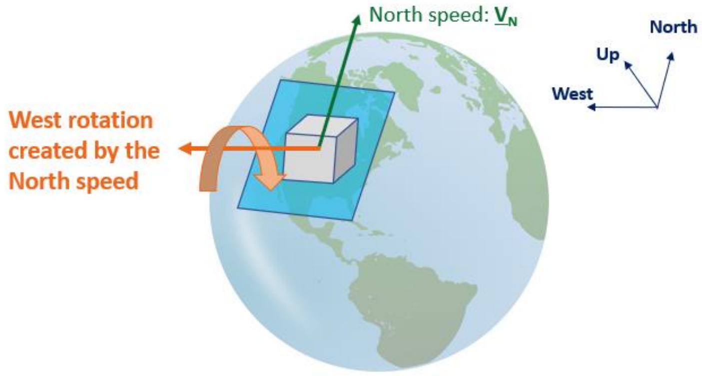

Figure 8: A North speed creates a Westward rotation

Consequently, wrongly assuming that the speed is null will lead to a heading error. The impact of the North speed on the heading is

$$\omega_E^{NS}\left[\frac{rad}{sec}\right] = \frac{V_N}{R_0} \qquad \text{Eq. 18}$$

With:

- ω_E^{NS}: [rad/sec] Rotation rate in the East direction caused by a North speed
- V_N: [m/sec] North speed
- R_0: [m] Earth radius at the equator. R_0 is about 6370km. In reality, since the Earth is an ellipsoid and not a sphere, one should use the local curvature at the system position, but this effect can be neglected[2].

Converting this formula to °/h:

$$\omega_E^{NS}[°/h] = 3600.\frac{180}{\pi}.\frac{V_N}{R_0} \qquad \text{Eq. 19}$$

This rotation, if not compensated, will have the same effect than a gyro bias on the heading accuracy. This means that the formula used to evaluate the gyro bias impact on the heading error can be reused: $\delta H[°seclat] \approx 4.b_g^r[°/h]$.

Consequently:

$$\delta H[°seclat] = 4.3600.\frac{180}{\pi}.\frac{V_N}{R_0} \qquad \text{Eq. 20}$$

Reducing the numeric parameters:

[2] Since the true Earth radius varies from 6357 km to 6378 km, that means a variation of up to 0.2% from 6370 km, the formula is correct up to 0.2%.

Impact of the speed error on the heading precision when the system is considered static	Eq. 21
$\delta H[°seclat] = 0.13. V_N[m/sec]$	

Limitation

The method described here includes an important limitation: in addition to very high performing sensor, one needs to be completely static, which is not always possible in real life.

Numerical application: If the alignment procedure takes 1 minute, and the system moves from 0.01° during that time, the corresponding rotation error is 0.01x60 = 0.6°/h, the heading error will be the same as a gyro bias of 0.6°/h (that means 2.4 ° seclat).

Since 0.01° corresponds to 20 micrometers displacement for a system with a 20 cm length, it is obvious that in real life, the conditions to directly measure the Earth rotation are almost never met.

However, if a direct measure of the Earth rotation is not feasible in real-life, an indirect measurement can be determined by using the derivative of the acceleration. This methodology is described below, but all the formula of this paragraph are still applicable because in both cases the idea is to measure the Earth rotation using the gyros.

USING THE ACCELERATION DERIVATIVE

Conventions

In this paragraph, the frames defined in Figure 9 below will be used:

- [b]: Body frame: Orthonormal frame linked to the IMU, with axis in the forward, left and up direction.
- [i]: Inertial frame. Static frame with respect to the stars. The inertial frame is usually chosen as the [b] frame at a certain time
- [n]: Navigation frame. In this document, it is defined as [North, West, Up]

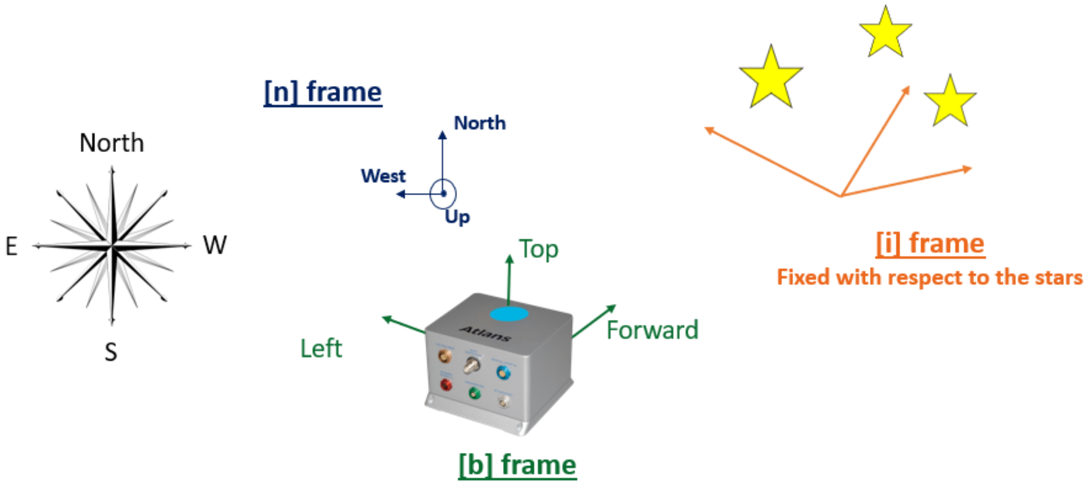

Figure 9: Frames definition

The following conventions are also used:

- For any vector $\underline{X}$, a subscript is used to defined in which frame the vector is projected. For example, $\underline{X}_n$ defines the coordinate of vector $\underline{X}$ in frame [n].
- Also, for any rotation vector $\underline{\omega}$, a superscript is used to define the frames that are rotating with respect to each other. For instance, $\underline{\omega}^{b/i}$ represents the rotation of frame [b] considering the frame [i] static.

How it works

An alternative method to measuring the Earth rotation, which is very small, is to use the gravity. As a matter of fact, the gravity is about 10m/sec², which is easy to measure.

To do so, the idea is to use the derivative of the acceleration in the inertial frame [i].

In static conditions:

- the derivative of the acceleration considering the inertial frame [i] to be static, projected in the [n] frame, is given by the Coriolis formula: $\left\{\left[\frac{d\underline{a}}{dt}\right]_i\right\}_n = \underline{\omega}_n^{n/i} \times \underline{a}_n$
- the only acceleration measured by the accelerometers is the gravity reaction: $\underline{a}_n = -\underline{g}_n$

A comment about acceleration measurements
When static on the Earth surface, an accelerometer does not measure 0, it measures the gravity reaction.
As a matter of fact, by application of Newton's second law of motion, an inertial sensor would measure 0 only in free fall conditions that means when it would not be accelerating in the inertial frame).
In static condition:

$$\underline{a}_n = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

Without any loss of generality, it is possible to consider that the [b] and [n] (North, West, Up) frames are identical (if they are not, the same results apply: this can be demonstrated using a constant transform matrix between the [b] and [n] frames).

In that case:

$$\underline{\omega}_{n}^{n/i} = \begin{bmatrix} \Omega_E.\cos(\text{lat}) \\ 0 \\ \Omega_E.\sin(\text{lat}) \end{bmatrix} \qquad \text{Eq. 22}$$

Thus, the derivative of the acceleration is:

$$\left\{\left[\frac{d\underline{a}}{dt}\right]_i\right\}_n = \begin{bmatrix} \Omega_E.\cos(\text{lat}) \\ 0 \\ \Omega_E.\sin(\text{lat}) \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \qquad \text{Eq. 23}$$

$$\left\{\left[\frac{d\underline{a}}{dt}\right]_i\right\}_n = \begin{bmatrix} 0 \\ -\Omega_E.\cos(\text{lat}).g \\ 0 \end{bmatrix} \qquad \text{Eq. 24}$$

Consequently, the direction of the acceleration derivative is oriented toward the East. If necessary, the North direction is easily found using the cross product of the acceleration derivative (oriented toward East) and the gravity.

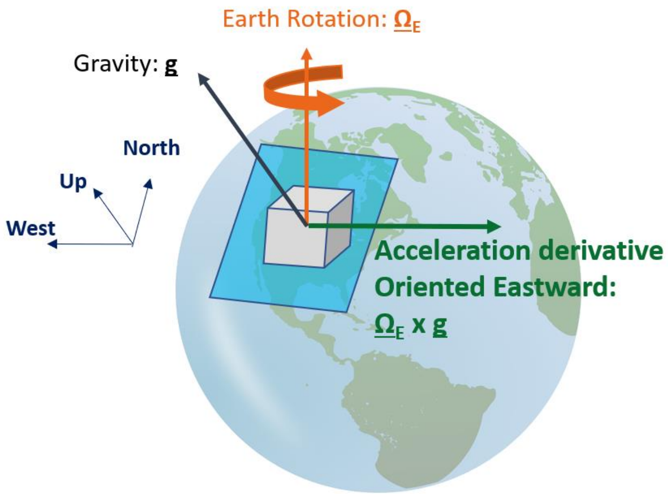

Figure 10: The East direction is provided by the derivative of the acceleration

A simple way to compute the East direction is to integrate the acceleration in the inertial frame. The inertial frame is defined to be the body frame at startup, so integrating the gyros allows computing the transform matrix to go from the [b] frame to the [i] frame at any time.

Suppose now that there is an error in the heading estimated by the INS. The [n] frame used by the INS is not the real [n] frame, because of the heading error. Let's note $[\hat{n}]$ the estimated [n] frame used by the INS.

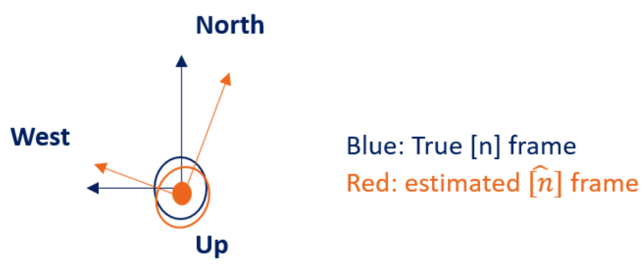

Figure 11: True Vs Estimated navigation frames

Let's note $\underline{\hat{\omega}}_n^{n/i}$ the Earth rotation projected in the $[\hat{n}]$ frame used by the INS:

$$\underline{\hat{\omega}}_n^{n/i} = \begin{bmatrix} \Omega_E.\cos(\text{lat}).\cos(\delta H) \\ -\Omega_E.\cos(\text{lat}).\sin(\delta H) \\ \Omega_E.\sin(\text{lat}) \end{bmatrix} \quad \text{Eq. 25}$$

To the first order in δH:

$$\underline{\hat{\omega}}_n^{n/i} = \begin{bmatrix} \Omega_E.\cos(\text{lat}) \\ -\Omega_E.\cos(\text{lat}).\delta H \\ \Omega_E.\sin(\text{lat}) \end{bmatrix} \quad \text{Eq. 26}$$

As it is clear from Figure 11, the INS behaves as if it were measuring $\underline{\omega}_n^{n/i}$, whereas in reality it measures $\underline{\hat{\omega}}_n^{n/i}$.

Consequently, the INS can compute the theoretical versus the measured acceleration derivative.

If $\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n$ is the error on the acceleration derivative, such that:

$$\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n = \left\{\widehat{\left[\frac{da}{dt}\right]_i}\right\}_n - \left\{\left[\frac{da}{dt}\right]_i\right\}_n \quad \text{Eq. 27}$$

Then:

$$\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n = \left(\begin{bmatrix} \Omega_E.\cos(\text{lat}) \\ -\Omega_E.\cos(\text{lat}).\delta H \\ \Omega_E.\sin(\text{lat}) \end{bmatrix} - \begin{bmatrix} \Omega_E.\cos(\text{lat}) \\ 0 \\ \Omega_E.\sin(\text{lat}) \end{bmatrix}\right) \times \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad \text{Eq. 28}$$

$$\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n \approx \begin{bmatrix} -\Omega_E.\cos(\text{lat}).\delta H.g \\ 0 \\ 0 \end{bmatrix} \quad \text{Eq. 29}$$

With:

- $\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n$ is the error on the acceleration derivative
- Ω_E: [rad/sec] Earth rotation rate
- lat: [rad] Latitude
- δH: [rad] Heading error
- g: [m/sec^2] Gravity

By measuring this difference, one can directly deduce the heading error from the gyros and accelerometers measurements.

<u>Notice: What makes this method very useful?</u>
In this paragraph, it has been assumed that the system was static. So why is that different from the former methods? Because in reality, the static hypothesis is not required: both the rotations and the linear displacement can be compensated:

- Compensating the rotations

If the system is rotating, the rotations can be compensated using the gyro measurement. So, although it's practical to consider the system static to understand what's going on, there is no real need for that.

- Compensating the displacements

The static requirement is not a requirement: if the system moves, the same principles apply, the only difference will be that the computed error $\delta\left\{\left[\frac{da}{dt}\right]_i\right\}_n$ will not be zero; but if the INS knows the true movements (thanks to a GNSS for instance), the theoretical value can be computed, and the heading estimated.
In other words, using the static hypothesis is a very practical simplification that allows determining the theoretical formula relating the measurement errors to the heading performance, and this formula can be generalized to non-static alignments.

Performance evaluation

A bias error or a North speed error will have the same impact as using the gyro measurement directly, because they will create an error in the transform matrix.

With this method, in addition to the gyro errors, an error on the measured derivative of the acceleration in the North direction will also have an impact on the heading accuracy. We can illustrate with a simple example: let's consider an INS with the forward axis perfectly aligned with the North direction. Let's also suppose that the bias of the forward accelerometer varies with time (put it another way: its bias derivatives is not null). In this scenario, the accelerometer bias derivative will create an error in the measured acceleration measurement, that will in turn create a heading error, as can be seen in Figure 12.

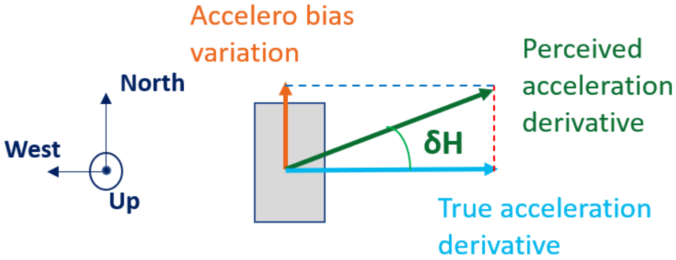

Figure 12: Impact of an accelerometer bias variation

Consequently, a variation of the accelerometer bias (often occurring at startup) will create an error of:

$$\delta H[\text{rad}] = \frac{\dot{b_N^a}}{\Omega_E.\cos(\text{lat}).g} \quad \text{Eq. 30}$$

With:

- δH: [rad] Heading error due to the accelerometer bias derivative
- $\dot{b_N^a}$: [m/sec[3]] North accelerometer bias derivative
- Ω_E: [rad/sec] Earth rotation rate ($\Omega_E \approx 7.10^{-5}\text{rad/sec}$)
- lat: [rad] Latitude
- g: [m/sec[2]] Gravity ($g \approx 10\text{m/sec}^2$)

Doing the numerical application, and converting the heading rad to degrees lead to:

Accelerometer derivative bias impact on the heading precision	
$\delta H[°\text{seclat}] \approx 8.5 \times 10^4.\dot{b}_a$	Eq. 31

Numerical application: a warmup of 1 µG/sec will create a heading error of 0.85°.

The impact of an accelerometer bias on the formula can be evaluated starting back from the formula:

$$\left[\frac{da}{dt}\right]_i = \begin{bmatrix}\Omega_E.\cos(\text{lat})\\0\\\Omega_E.\sin(\text{lat})\end{bmatrix} \times \begin{bmatrix}0\\0\\g\end{bmatrix} \quad \text{Eq. 32}$$

If this measure is impacted by a constant accelerometer bias, it becomes:

$$\widehat{\left[\frac{da}{dt}\right]_\imath} = \begin{bmatrix}\Omega_E.\cos(\text{lat})\\0\\\Omega_E.\sin(\text{lat})\end{bmatrix} \times \begin{bmatrix}b_N^a\\b_W^a\\g\end{bmatrix} \quad \text{Eq. 33}$$

With:

- $\widehat{\left[\frac{da}{dt}\right]_\imath}$: [m/sec[3]] Measured derivative of the acceleration considering the inertial frame to be constant
- Ω_E: [rad/sec] Earth rotation rate ($\Omega_E \approx \frac{7.10^{-5}\text{rad}}{\text{sec}}$at: [rad] Latitude
- g: [m/sec[2]] Gravity ($g \approx 10\text{m/sec}^2$)
- b_N^a and b_W^a: [m/sec[2]] Northern and Western accelerometer biases

$$\widehat{\left[\frac{da}{dt}\right]_\imath} = \begin{bmatrix}-\Omega_E.\sin(\text{lat}).b_W^a\\-\Omega_E.\cos(\text{lat}).g + \Omega_E.\sin(\text{lat}).b_N^a\\\Omega_E.\cos(\text{lat}).g\end{bmatrix} \quad \text{Eq. 34}$$

To the first order:

$$\delta H[\text{rad}] = \frac{-\Omega_E.\sin(\text{lat}).b_W^a}{-\Omega_E.\cos(\text{lat}).g} \quad \text{Eq. 35}$$

Or, simplifying and converting the heading error to degrees seclat and the accelerometer bias to mG (which is the unit largely used to define an accelerometer bias):

Accelerometer Western bias impact on the heading precision

$$\delta H[°\text{seclat}] = 0.06.\, b_W^a[\text{mG}].\sin(\text{lat}) \quad \text{Eq. 36}$$

Numerical application: for a 1 mg bias the heading error will be 0.01° seclat. So this bias error has little impact on the heading error.

HEADING DETERMINATION DURATION

The alignment duration mainly depends on 2 factors:

- True movements
- Gyro noise (Angular Random Walk)

True movements

As explained in the former section, the basic logic behind the heading determination using the accelerometers is as follows:

- We start with a hypothesis for the true heading value
- An error on this value will create an error in the acceleration derivative
- This in turn will create an error on the speed, and consequently on the displacement
- If we know that the displacement is null (static hypothesis), we can recover the heading error from the displacement computed by the system

For some applications, the static hypothesis is not completely true. For instance, on a boat, we can have a speed because the boat is drifting. In that case, the system will be able to determine the heading only when the displacement wrongly computed because of the heading error becomes higher than the true displacement.

Eq. 29 showed that a heading error will create an error on the North acceleration derivative of $-\Omega_E.\cos(\text{lat}).\delta H.\hat{g}$ m/sec^3.

Considering that the latitude and the heading error are constant (which is true over a short period of time), one can integrate this effect three times to evaluate the impact on the position:

$$\delta\{P_i\}_n = -\frac{1}{6}\Omega_E.\cos(\text{lat}).\delta H.g.t^3 \quad \text{Eq. 37}$$

Numerical application: for a system at latitude 45°, a heading error of 0.1° will create a position error of 1.5 cm after 100 sec, and 15 meters after 1000 sec. This means that an external position sensor with cm-level accuracy is required to determine the heading with a ~0.1°-level accuracy after 100 sec; a positioning sensor with 15 meters accuracy can reach the same accuracy after 1000 sec.

Gyro noise (Angular Random Walk)

In static conditions, the heading performance mainly depends on the gyro performance, as seen in Eq. 17:

$$\delta H[°seclat] \approx 4.b_g^i[°/h]$$

In reality the gyro bias is not the only error: the gyro measurements are also impacted by the Angular Random Walk (ARW), as explained in the first Section of this document.

To cancel out the noise, the measurements can be averaged over a certain period t. The impact of both errors is evaluated as:

$$\begin{cases} \delta H^{noise}.t = 4.ARW[°/h/\sqrt{Hz}] \times \sqrt{t} \\ \delta H^{bias}.t = 4.b_g \times t \end{cases} \quad \text{Eq. 38}$$

With:

- δH^{noise}: [°] Heading error due to the ARW
- δH^{bias}: [°] : Heading error to the bias
- ARW: $[°/h/\sqrt{Hz}]$ Gyro angular random walk (notice that the ARW is usually provided in °/sqrt(h) and not in [°/h/sqrt(Hz)]. Here the unit $[°/h/\sqrt{Hz}]$ is used because the time t is in sec.
- b_g: [m/sec²] Accelerometer bias
- t: [sec] time

Dividing by t and converting the ARW from $[°/h/\sqrt{Hz}]$ to $[°/\sqrt{h}]$ gives:

$$\begin{cases} \delta H^{noise} = \frac{4}{60}.\frac{ARW[°/\sqrt{h}]}{\sqrt{t}} \\ \delta H^{bias} = 4.b_g \end{cases} \quad \text{Eq. 39}$$

Using those equations, the evolution of the heading accuracy can now be estimated:

<u>Impact of the gyro ARW on the alignment time</u>

$$\delta H = 4.\sqrt{\left(\frac{60.ARW[°/\sqrt{h}]}{\sqrt{t}}\right)^2 + b_g^2} \quad \text{Eq. 40}$$

Figure 13 illustrates how the ARW impacts the convergence time.

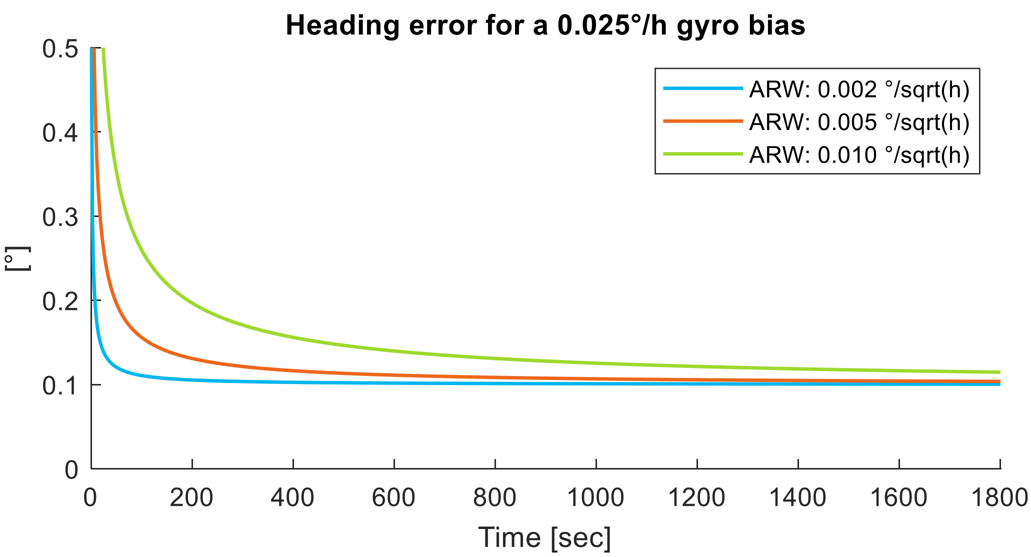

Figure 13: Heading accuracy convergence time

Synthesis

The following table provides a synthesis of the impact of the errors on the heading for a static alignment. Notice the difference made between the "truly static alignment" (the system does not move at all) and the multi-static alignment (the system goes through a procedure with various static periods at different angular positions).

Source of error	**Impact on the heading**
East gyro bias repeatability (truly static case)	$\delta H[°seclat] \approx 4.b_E^{gr}[°/h]$
East gyro bias instability (multi-static case)	$\delta H[°seclat] \approx 4.b_E^{gr}[°/h]$
North speed	$\delta H[°seclat] = 0.13.V_N[m/sec]$
West accelerometer bias	$\delta H[°seclat] = 0.06.b_W^{ar}[mG].\sin(lat)$
North accelerometer bias derivative	$\delta H[°seclat] \approx 8.5 \times 10^4.\dot{b}_N^a[m/sec^3]$

Table 1: Inertial sensors impact on heading performance

With:

- b_E^{gr}: West gyro bias repeatability
- V_N: North speed
- b_W^{ar}: West accelerometer bias repeatability
- $\dot{b}_N^a$: North derivative of the accelerometer bias

IV. Heading determination using external sensors

Two different techniques can be used to determine the heading using external sensors:

- The most obvious one consists of using the heading computed by another sensor. The is usually done using a magnetometer or a dual antenna GNSS, but in fact any other sensor can be used. Those solutions are described in the 2 first subsections below ("Measuring the magnetic field" and "Using an external geographic information")
- Another solution consists of comparing the integrated IMU data with an external sensor providing information about the displacement in the geographical frame. This is described in the last subsection: "Using displacements in the geographical frame"

MEASURING THE MAGNETIC FIELD

Since the magnetic field is oriented in the North/South direction, it is easy to measure using a magnetometer, and then to define the heading.

In reality the magnetic field is not exactly aligned with the North/South direction, but some organizations such as the NOAA (US National Oceanic and Atmospheric Administration) and the British Geological Survey provide map computing the deviation between the Geographical and Magnetic North at any place on Earth. The accuracy of such a map is typically a few hundredths of degrees but can reach a few degrees when unmodelled local abnormalities are present. Those maps are typically updated every 5 years [Ref2].

In addition to this error, some error due to the sensor degrades the performance. The magnetic field in the horizontal plane is about 0.25 to 0.65 Gauss, so the heading error based on the sensor performance is defined as:

$$\delta H[°] = \frac{180}{\pi} \frac{\text{Sensor bias}}{\text{Magnetic field}} \qquad \text{Eq. 41}$$

If the desired heading accuracy is 0.1°, with a magnetic field of 0.25 Gauss, a sensor with a bias repeatability of 0.46 mGauss (or 46nT, since 1nT=10^{-5} Gauss) or better must be used. This is much less than the accuracy of most sensors found in the industry.

Finally, another even more damaging source of error can be the magnetic interferences found in the nearby environment of the sensor. Any magnetic anomaly will create a heading error of the same amplitude as the sensor bias, as defined in Eq. 41:

$$\delta H[°] = \frac{180}{\pi} \frac{\text{Magnetic disturbances}}{\text{Magnetic field}} \qquad \text{Eq. 42}$$

Some disturbances originating from the vehicle are constant and can be compensated, but many cannot (engine running, metallic structures in the environment, etc.)

For all those reasons, especially the last one, magnetometers can be very convenient when the accuracy compared to true North or reliability are not very important, or when the environment is very well controlled (which does not happen often). Otherwise, they should be avoided.

USING AN EXTERNAL GEOGRAPHIC INFORMATION

It is easy to determine the heading of a line going through 2 features whose relative position is known, as illustrated in Figure 14:

$$H = \tan^{-1}\left[\left(\frac{\text{Long2} - \text{Long1}}{\text{Lat2} - \text{Lat1}}\right).\cos(\text{Lat})\right] \qquad \text{Eq. 43}$$

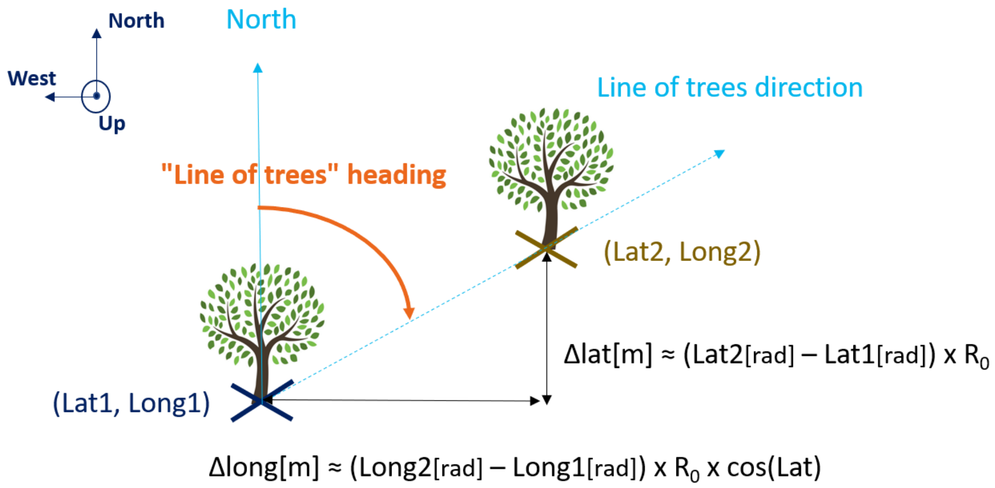

Figure 14: Heading of a direction defined by 2 positions - In this figure, the Earth is considered spherical with Radius R0, and that the difference of latitude between the 2 points is small. Lat = 0.5 x (Lat1+Lat2).

Consequently, if a sensor can determine the relative position of 2 features with respect to the INS orientation, then it is able to find the INS heading, as illustrated in Figure 15.

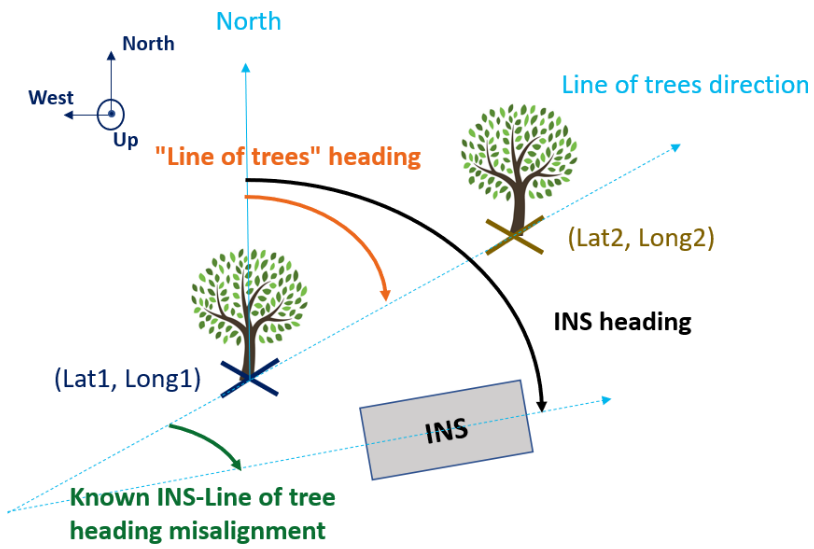

Figure 15: The heading of the INS can be determined using 2 features and the known misalignment between the features and the INS

As will become clear later, it is very important to note that what is needed is not the actual position of both elements: it is an accurate relative position (and an approximate real position).

GNSS double antenna

The most commonly used heading measurement system using this technology is the “double antenna GNSS”.

Eq. 43 can be used to determine the orientation of two GNSS antennas. Such a GNSS system, called dual antenna system, can be calibrated with respect to an INS, and then used to align this INS.

Figure 16: GNSS dual antenna system

What makes this solution interesting is that the absolute position of the system does not need to be accurate, only the relative position of both antennas. So even if the GNSS system has a meter level accuracy, the relative position of both antennas can be of a few millimeters.

It is straightforward to see that:

$$\delta\mathrm{H[deg]} = \frac{180}{\pi} \cdot \frac{\text{relative positioning error}}{\text{baseline length}} \qquad \text{Eq. 44}$$

With:

- $\delta\mathrm{H}$: [°] Heading error
- $\text{relative positioning error}$: [m] Difference of positioning error between the two antennas
- baseline length: [m] Distance between both antenna

Using the same principles as a RTK (Real-Time Kinematic) GNSS, a dual antenna system can compute a relative position with an error of a few millimeters.

Suppose for instance that two antennas are separated by 1 meter and the relative position accuracy is 5 mm, the heading accuracy will be:

$$\delta\mathrm{H[deg]} = 0.3°$$

From Eq. 44, it is clear that the longer the baseline, the better the accuracy. Notice though that both antennas should be rigidly attached, which is very difficult on a large platform when its moving.

In the absence of high-grade gyro, the double antenna system is very efficient to determine the heading. The following limitation must be kept in mind:

- To have a very accurate heading position, one needs a long baseline, which is not necessarily possible on a vehicle.
- When the GNSS is lost or the reception is bad (as in a city center), the heading is also lost, making it a good complement to an INS, but not necessarily a good substitute.

Perceptive sensor

Another way to determine the heading is to use a perceptive sensor such as a LiDAR or a stereo camera, and to compute the heading using the previously known location of some features (i.e.: using a feature map).

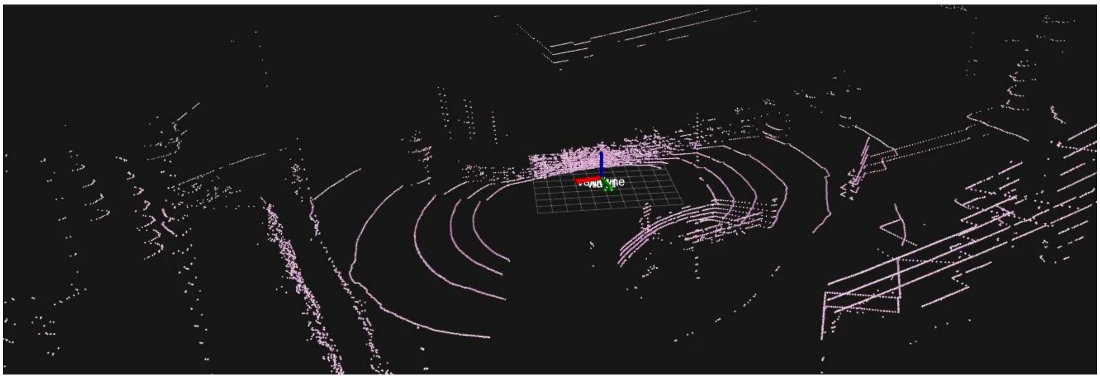

Figure 17: Example of LiDAR image

As for the GNSS dual antenna, the heading accuracy is directly linked to the accuracy of the perceptive sensor and the map.

USING DISPLACEMENTS IN THE GEOGRAPHICAL FRAME

Acceleration measurement

It was explained earlier that the accelerometers can be used to determine the heading by indirectly measuring the Earth rotation direction.

However, there is another way to use the accelerometers, it is to… accelerate. If a system accelerates, the INS will measure the acceleration, and project it on the Geographical frame to compute the geographical speed and position.

If the heading is incorrect, the projection will be incorrect and thus the acceleration will create a speed error in the transverse direction of the movement.

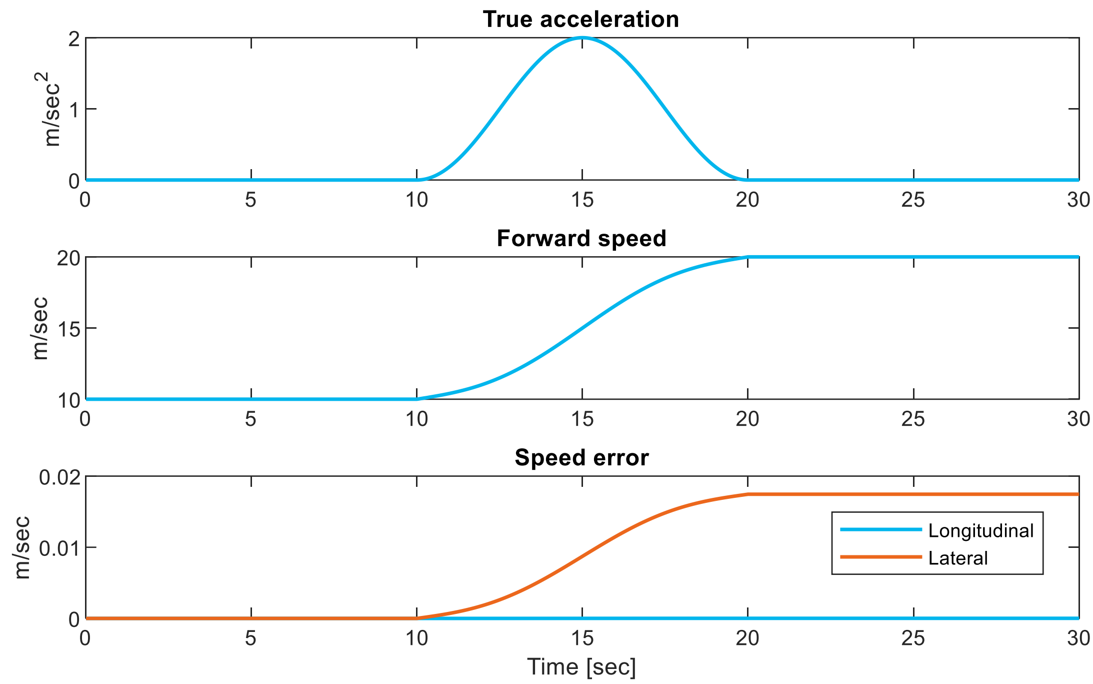

Figure 18: Impact of the heading error on the speed in case of acceleration - The image illustrates the impact of a 0.1° heading error. When the system accelerates from 10 to 20m/sec, a lateral speed of 0.017m/sec is created in the lateral direction (i.e., the direction orthogonal to the acceleration)

For a short period of time [t0-t1], the heading error can be considered constant, and thus:

$$\delta V_{Geo}(t1) = \delta H. \int_{t0}^{t1} a.dt \tag{Eq. 45}$$

$$\delta P(t1) = \delta H. \iint_{t0}^{t1} a.dt \tag{Eq. 46}$$

This error can be measured and corrected using an external position sensor such as a GNSS, and, thanks to the data fusion algorithm, the heading error can be estimated.

Notice that a speed sensor can also do the same thing, with one caveat: it needs to be a sensor providing the geographical speed, not the body speed. Most speed sensors, such as DVL (Doppler velocity Measurement, used for underwater applications) or the DMI (Distance Measuring Instrument, used on wheeled vehicles) provide the body speed. This means they cannot help estimating the heading.

It is not easy to define the heading accuracy one can achieve using the acceleration, because it depends on various factors, such as the external sensor accuracy, the accelerometer noise, the gyro noise (because the gyro noise will create a roll/pitch noise, which will in turn create a horizontal acceleration noise due to the gravity projection error). The following figures provide some examples in various conditions.

Let's take the example of the following trajectory: a system is still for 100 sec, and then accelerate to a certain speed in 25 seconds, and then go straight at a constant speed.

Figure 19: Dynamic profile of the test trajectory: forward speed

To evaluate the impact of a parameter, it is useful to evaluate the heading convergence time on a baseline situation, and then modify the value of the parameter of interest. The baseline is realized with the following parameters:

- GNSS: 3cm accuracy
- Gyro white noise: 0.1°$/\sqrt{\mathrm{h}}$
- Accelerometer white noise: 100 $\mu\mathrm{G}/\sqrt{\mathrm{Hz}}$
- All simulations start with a heading standard deviation of 1.5° (the initial value does not impact the convergence time, using a small value allows easier interpretation of the figures)

Gyro noise impact

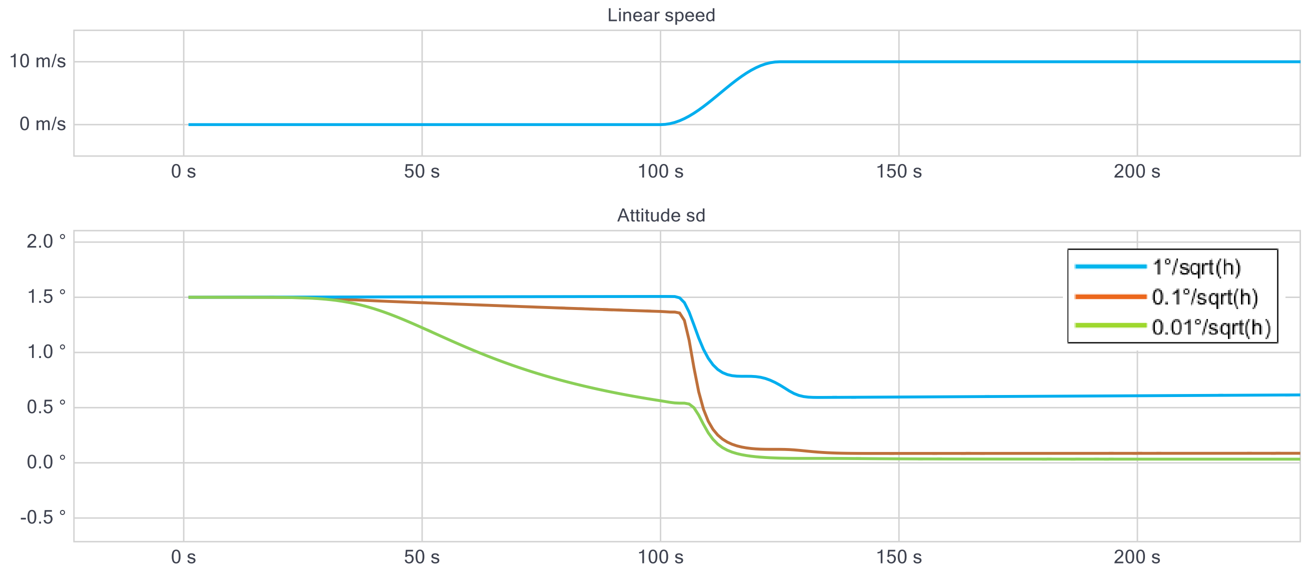

Figure 20: Heading standard deviation with varying gyro noise

Figure 20 shows that with a gyro noise of 1°$/\sqrt{\mathrm{h}}$, the acceleration is not sufficient to accurately estimate the heading. However, there is no big difference between 0.1°$/\sqrt{\mathrm{h}}$ and 0.01°$/\sqrt{\mathrm{h}}$: this is because in both cases the gyro noise is not the main source of error in the position. Improving the gyro noise will only make a difference up to a certain value.

It is also interesting to note that, once the final speed is reached, the heading estimation is not improved. This is because the heading cannot be estimated on a straight line.

The reason is that a heading error with a constant speed will create an error in the geographical frame, not in the body frame. If the only aiding sensor available is a speed sensor defined in the body frame, a heading error will not create any inconsistency between the speed computed by the INS and the speed measured by the aiding sensor.

Accelerometer noise impact

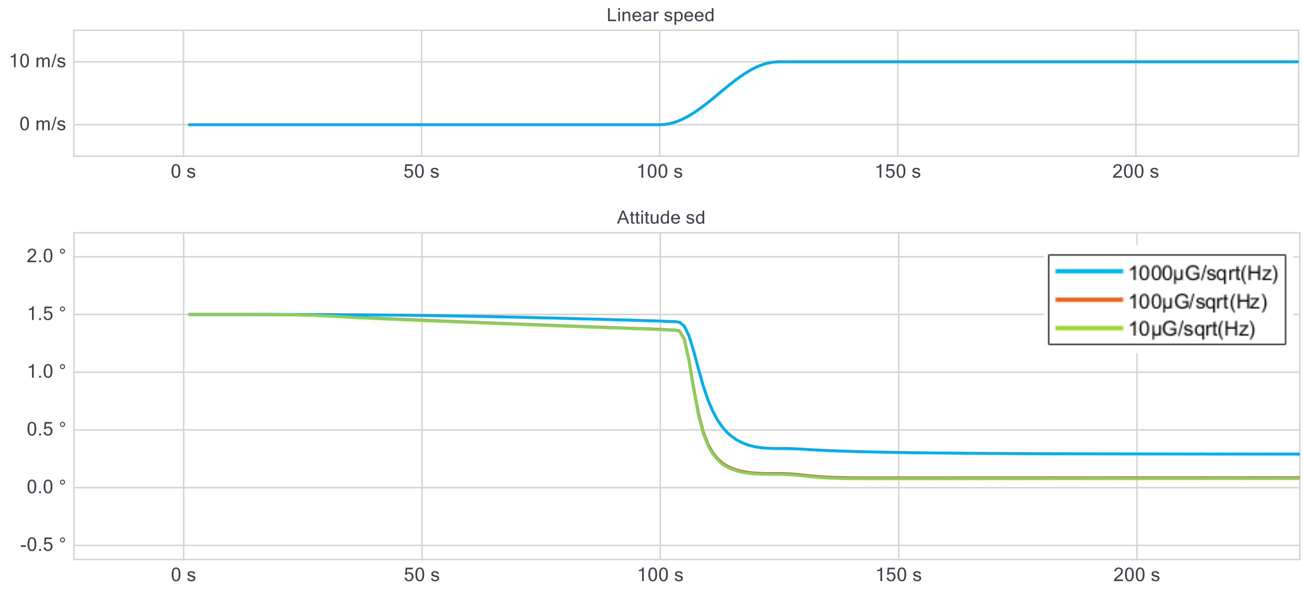

Figure 21: Heading standard deviation with varying gyro noise

As with the gyro, a lower accelerometer noise leads to a better heading performance, but only up to a certain point: there is no difference between a noise of 100 µG/sqrt(Hz) and a noise of 10µG/sqrt(Hz) (the 2 lines are superimposed on the figure).

GNSS accuracy

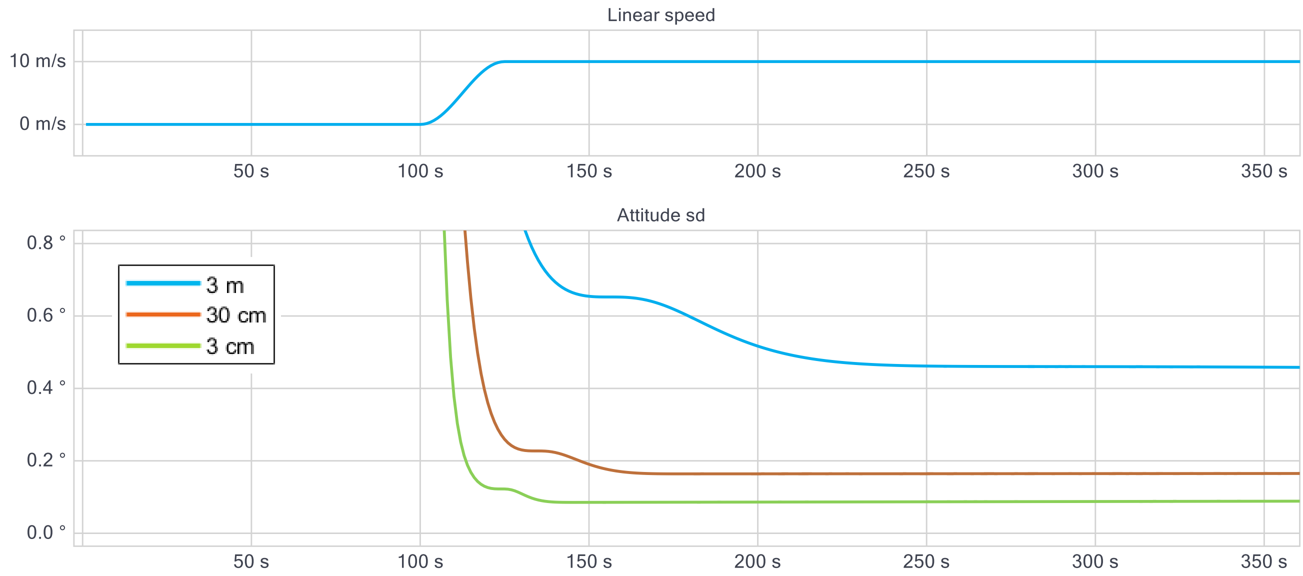

Figure 22: Heading standard deviation with varying GNSS noise

The figure above shows that the GNSS accuracy has a very strong impact both on the convergence time and the final accuracy of the heading.

Acceleration

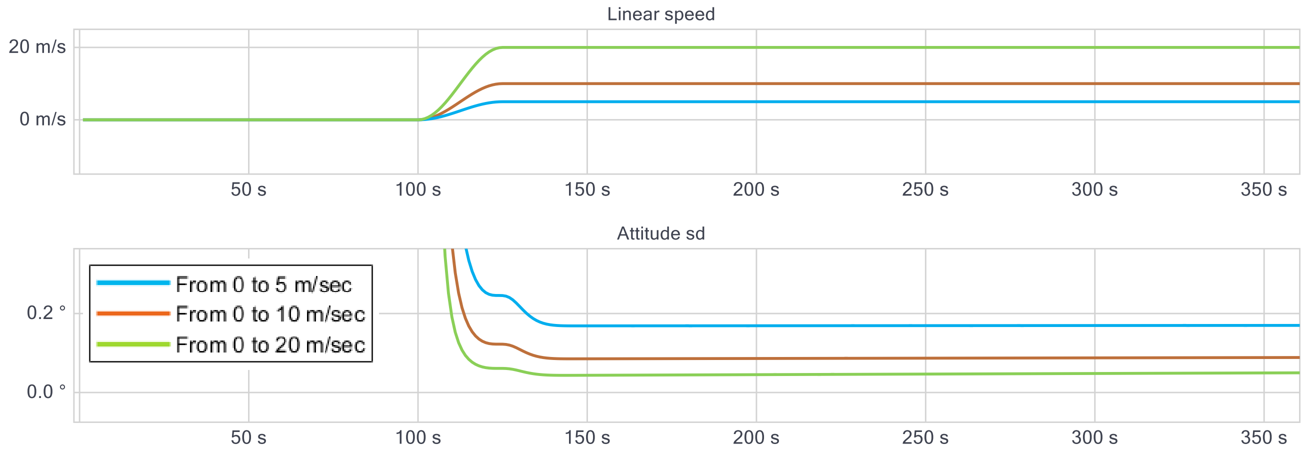

Figure 23: Heading standard deviation with varying acceleration

The final heading accuracy is proportional to the acceleration. To be more accurate: the acceleration is not what matters: it is the amplitude of the variation in speed value.

Speed measurement

As explained before, using a sensor measuring the speed in the body frame does not allow estimating the heading. However, using such sensor and a positioning sensor can solve the problem as illustrated below.

Using the same reasoning as for the acceleration:

$$\delta P(t1) = \delta H. \int_{t0}^{t1} V_b^H . dt \qquad \text{Eq. 47}$$

With:

- $\delta P(t1)$: [m] Position error at t1
- V_b^H: [m/sec] norm of the horizontal component of the body speed

Consequently:

$$\delta H[rad] = \frac{\delta P(t1)}{\int_{t0}^{t1} V_b^H . dt} \qquad \text{Eq. 48}$$

Contrary to the previous case, using the speed sensor instead of the acceleration allows estimating the heading even in straight lines.

Suppose for instance a system has a constant speed V of 10m/sec, and an external position sensor with 3 cm accuracy is used. The INS will be able to estimate the heading error with an accuracy of:

$$\delta H[°] = \frac{180}{\pi} . \frac{0.03}{10.t} \approx \frac{0.18}{t} \qquad \text{Eq. 49}$$

This formula shows that in these conditions, the heading error is very observable. In theory, a heading accuracy better than 0.01° can be achieved in less than 20 sec.

In reality, the performance will be limited by the performance of the gyros (again!) and the external sensors accuracy.

To illustrate this, let's use a trajectory with constant speed 10m/sec, and the same baseline that was used before:

- GNSS: 3 cm accuracy
- Gyro white noise: 0.1$°/\sqrt{\mathrm{h}}$
- Accelerometer white noise: 100 $\mu\mathrm{G}/\sqrt{\mathrm{Hz}}$
- All simulations start with a heading standard deviation of 1.5° (the initial value does not impact the convergence time, using a small value allows for figures easier to interpret)

The following figures illustrate the impact of some parameters on the heading accuracy.

Gyro noise impact

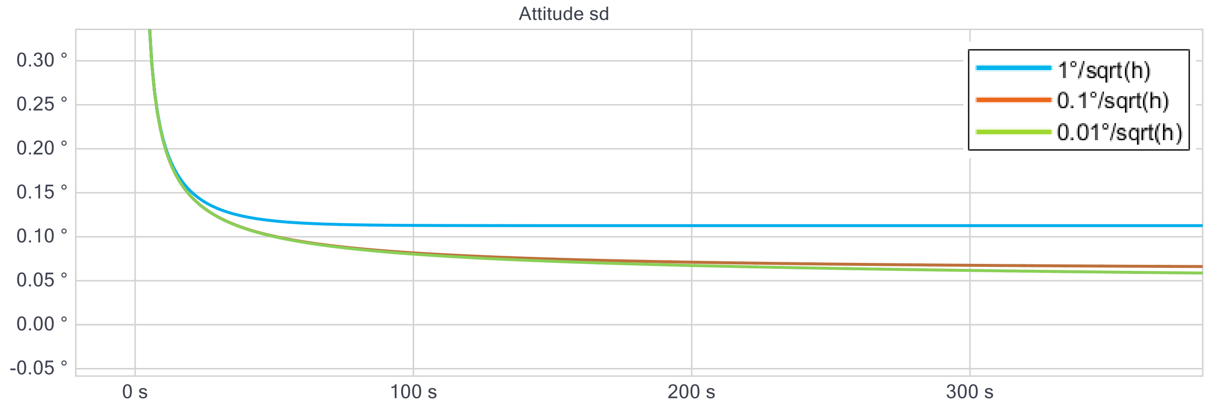

Figure 24: Heading standard deviation with varying gyro noise

As for the acceleration, lowering the gyro noise allows for a better heading, but only up to a certain point.

Speed sensor accuracy impact

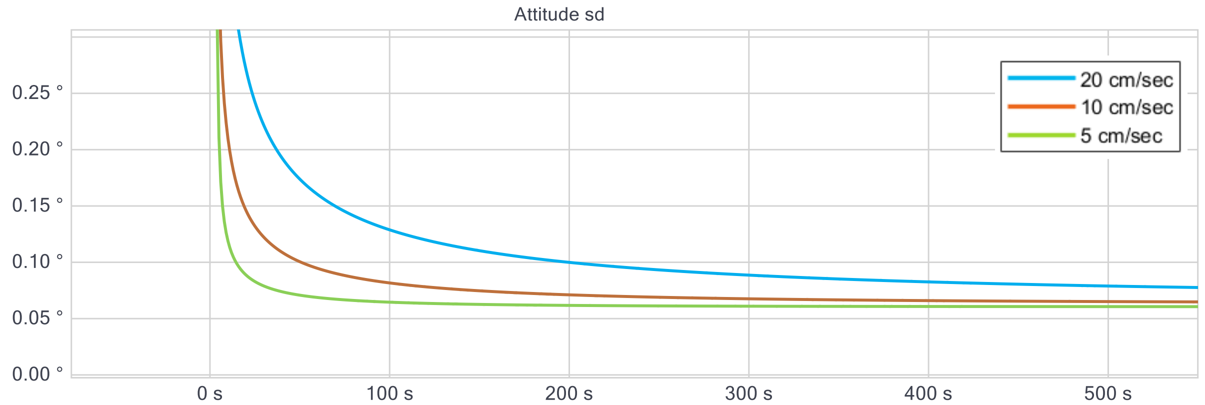

Figure 25: Heading standard deviation with varying gyro noise

With time, the heading can be computed with approximately the same accuracy in the three cases, because the sensor noise can be averaged. But the alignment is faster when speed accuracy is improved.

GNSS accuracy

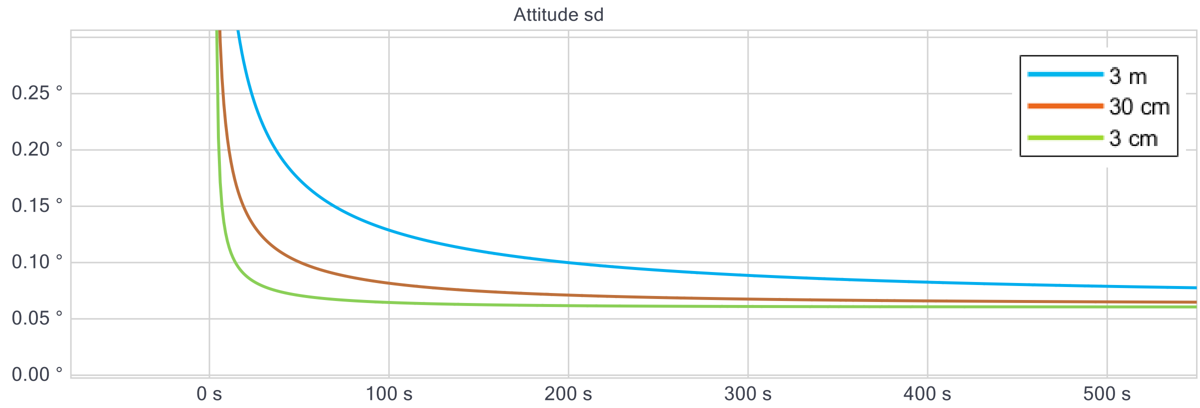

Figure 26: Heading standard deviation with varying GNSS noise

It is clear from Figure 26 that the GNSS accuracy has a major impact on the convergence time.

V. Conclusion

The alignment is the process by which the initial conditions of an INS are determined. Those initial conditions include at least the initial position, speed, and orientation of the system; they can also include some inertial sensor parameters (such as the gyro or accelerometers bias or scale factor).

This process is extremely important as it will have a critical impact on the performance of the system during the whole navigation.

As this paper showed, defining the right alignment procedure depends on various parameters, especially the aiding sensors available (GNSS, speed sensor, etc.) and their precision, the performance of the inertial sensors, and the dynamic of the vehicle during the alignment phase. Yet, some general guidelines can be drawn:

- The position comes either from a position sensor, such as a GNSS or an USBL (Ultra-Short Baseline) for subsea applications, or is provided by the user.
- The speed is either deduced from a position sensor, computed from a speed sensor, or defined by the user when the system is static.
- The roll and pitch are computed using the horizontal accelerometers.
- The heading is the most complicated parameter to estimate, and many solutions exist to compute it. The following table provides a comparative overview of the different solutions.

Solution	**Pros**	**Cons**
Measuring the Earth rotation using the inertial sensors only	• Only solution to find the heading in any circumstances • Accuracy	• High performing gyros are required (typically: FOG or RLG)
Measuring the magnetic field using a magnetometer	• Small • Low cost	• Bad accuracy • Unreliable, depend on the magnetic environment
Using an external geographical information: Dual antenna GNSS	• Ratio price/Performance • Fast alignment	• Only works where the GNSS reception is good • Long baseline (i.e. not possible for small drone application for instance) • Heading baseline harmonization needed
Using an external geographical information: Perceptive system	• Cost efficient	• Accurate map required
Measuring the displacement: Acceleration measurement	• Cost efficient	• Require some movements • Require some correct and accurate position sensor reception
Measuring the displacement: Speed measurement	• Cost efficient	• Require some movements • Require some correct and accurate position sensor reception • Requires a 3D velocity sensor

Table 2: Heading determination solutions

Notice that this document focused on the physical principles underlying the heading determination. This is very helpful to understand the intrinsic limitation of each of them. However, this does not mean that an ad-hoc algorithm must be design for each solution: a well-designed algorithm can take any input from any sensor, and provide the optimal performance discussed above.

VI. Appendix

Frame definition

In this document, various frames are used.

[b] Body frame: Orthonormal frame linked to the IMU, with axis in the forward, left and up direction.

[i] Inertial frame. Static frame with respect to the stars. The inertial frame is usually chosen as the [b] frame at a certain time

[n] Navigation frame. In this document, it is defined as [North, West, Up]

VII. References

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